

REINFORCING THE UNDERSTANDING OF PLACE VALUE THROUGH NUMBER BASES

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The purpose of this workshop is to give ideas on Using Number Bases to reinforce the understanding of Place Value. It is well known that in many disadvantaged schools, learners perform at a level much lower than the Grades in which they are studying. This can place a huge challenge for teachers to close existing learning gaps in learners, considering the quantity of work that has to be covered each year. If these gaps are not addressed early enough, learners progress to subsequent grades with even greater gaps in higher classes.

*Although place value is taught up to Grade 6 in the curriculum, learners in Grade 7, 8 even 9 tend to struggle with place value related concepts such as rounding off, and working with the four operations, among others. Whilst solidifying the understanding of Place value may improve the understanding of such concepts, it can also assist in the practice of exponents and finding the general term of a sequence. The activities in this workshop could be used by teachers as **expanded opportunities** for learners in the senior phase who struggle with the place value concept.*

TARGET AUDIENCE:	Senior Phase
DURATION:	One Hour
NUMBER:	Not more than 50.

MOTIVATION FOR THIS WORKSHOP

Most learners get to Grade 7, 8 and 9 still lacking in the basics of place resulting in their inability to access higher concepts that require a deep understanding of place value. Place value is taught up to Grade 6 and by this time they should be fluent in their four operations calculations. However, this is not the case and it makes it difficult for teachers to simply teach at the level of the Grade.

If teachers were to simply give them exercises in basic concepts, learners will be bored and undermined. The activities designed here will give the learners adequate challenge yet simple concepts to practice lower concepts. The activities can be used as an intervention strategy at extra time rather than do it in class.

Teachers will benefit by receiving materials that are readily available for use.

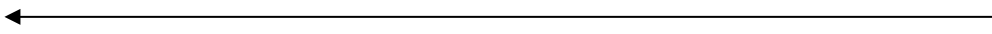
HOW CAN BASES REINFORCE THE KNOWLEDGE OF PLACE VALUE.

Learners in senior phase may feel uncomfortable doing activities extracted purely from lower classes textbooks although lack of mastery of concepts learned at these lower classes are a huge hindrance to their progression in the learning of higher concepts. The use of activities such as the ones below affords them enough challenge for their age yet are easy enough to complete individually once the concepts have been taught to them.

Number bases

A **base** is a method of expressing numbers using place value which is the value of a digit based on its specific position. With the use of columns and a number pattern that will help in the understanding of the concept of bases can be formed.

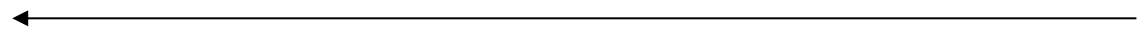
Firstly the column should always be read from the right to left; hence the first furthest column on the right always begins with one.



Position	6	5	4	3	2	1
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Starting Point

Using the base x , and raising the base x exponentially (these bases represent place values), the base at position 1 always starts with x^0 and exponents continue consecutively as the position increase consecutively simultaneously. Reading the column from right to left we get:



Position	6	5	4	3	2	1
Bases	x^5	x^4	x^3	x^2	x^1	$x^0 = 1$

Starting Point

Applying the above to a number, for example the number 123 in other words in base x , 123 means:

We write the number backwards from right to left starting from position 1.

Position	6	5	4	3	2	1
Bases	x^5	x^4	x^3	x^2	x^1	$x^0 = 1$
Number				1	2	3

Determining base x of 123 means:

$$123_x = (1 \times x^2) + (2 \times x^1) + (3 \times x^0)$$

Position	6	5	4	3	2	1
Bases	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
Number				1	2	3

Using the same number with base 2 (123 base 2):

$$\begin{aligned} \therefore 123_2 &= (1 \times 2^2) + (2 \times 2^1) + (3 \times 2^0) \\ &= 4 + 4 + 3 \\ &= 11 \end{aligned}$$

Base Five and Base Two numeration systems

In the decimal system, numbers are grouped in 10's whereas in base five they are grouped in 5's and grouped in 2's in base two. The binary system is mostly used in computer systems where the digits zero and one operate as "on (1)" and "off (0)" in the case of switches in simple terms.

Example: 28 in base 10 is 2 tens and 1 eight but in base five there are 5 fives

28₁₀
is:



TENS	UNITS
	
2	8

Fig 1 A representation of 28 in base 10

103₅
is:

TWENTY-FIVES	FIVES	UNITS
		
1	0	3

Fig 2 A representation of 28 in base 5

When you say the number, it reads, one, zero, three base five and **NOT** one hundred and three base five. It is important to emphasise that in base 10, 28 has TWO place values whereas it has THREE in base five. The two numbers represent 28 one in base 10 and the other in base 5. Every number in a particular base has all its digits less than the base. For example, the largest digit in base 5 is 4 and the largest digit in base 8 is 7.

Mendieta states that “The depth of conceptual understanding one has about a particular mathematical concept is directly proportional to one’s ability to translate and transform the representations of the concept across and within a wide variety of representational systems.” It is highly possible that with this in mind, the use of

knowledge of bases will increase the learners' understanding of place value as they represent numbers across different number bases.

Base Five and Base Two numeration systems

Activity 1: Make a representation of 28 for Base Two as shown above and write the number of how many place.

[5 min]

Place Values in other Bases:

Table 1: Converting 28 to base 2 and 5

The Number 28 in base 10 converted to base 5 and 2							
Base	Number	Place Value				No. of Places	
Base 10	28			10^1	10^0	2	
				10	1		
				10	1×8		
				$\times 2$			
				$20 + 8 = 28$			
Base 5	103_5			5^2	5^1	5^0	3
				25	5	1	
				25	5×0	1×3	
				$\times 1$			
				$25 + 0 + 3 = 103_5$			
Base 2	11100_2	2^4	2^3	2^2	2^1	2^0	5
		16	8	4	2	1	
		16	8×1	4×1	2×0	1×0	
		$\times 1$					
		$16 + 8 + 4 + 0 + 0 = 11100_2$					

The table above shows how to convert base 5 and 2 to base 10.

Another method to convert a number from base 10 to another base is shown below. This method can be introduced much later when the concept of place value is well grasped:

Table 2: Another method of converting 28 to base 2 and 5

Converting 28 base 10 to base 5 and 2			
28 in base 5		5	28
	$(28 \div 5)$	5	5 rem 3
	$(5 \div 5)$	5	1 rem 0
	$(1 \div 5)$	5	0 rem 1
In reverse order the answer is 103₅			
28 in base 2		2	28
	$(28 \div 2)$	2	14 rem 0
		2	7 rem 0
	$(14 \div 2)$	2	3 rem 1
	$(7 \div 2)$	2	1 rem 1
	$(3 \div 2)$	2	0 rem 1
	$(1 \div 2)$	2	0 rem 1
In reverse order the answer is 11100₂			

↑

READING IN REVERSE ORDER

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Converting other number bases to base 10.

Activity 2: Convert the following numbers (in base 10) below, to the base shown in the bracket.

- a) 103 (base 5)
- b) 76 (base 2)
- c) 46 (base 3)
- d) 245 (base 8)
- e) 105 (base 4)

[5 min]

The exercise below is for learners to convert numbers from other bases to base 10: It is encouraged that they always write the place values in exponential form and then as a value.

Activity 3: Write the following numbers in base 10.

- a) 1110011₂
- b) 4031₅
- c) 705₈
- d) 3230₄
- e) 1221₃

[5 min]

Addition and Subtraction of number bases other than 10

Activity 4: Subtract or add the following numbers.

- a) $1110011_2 - 1101110_2$
 b) $4031_5 + 4114_5$
 c) $705_8 - 656_8$
 d) $3230_4 + 3333_4$
 e) $1221_3 - 1122_3$

[5 min]

Examples:

Table 3: Illustration of addition or subtraction of two numbers with other bases

	64	32	16	8	4	2	1
a)	1	1	⁰ 1	¹ 0	² 0	1	1
	1	1	0	1	1	1	0
	0	0	0	0	1	0	1
Answer	101₂						
b)			625	125	25	5	1
	¹			4	¹ 0	¹ 3	1
				4	1	1	4
	1 3 2 0 0						
	13200₅						
CHECKING THE ANSWERS BY CONVERTING ALL NUMBERS TO BASE 10							
a)	64	32	16	8	4	2	1
	1	1	1	0	0	1	1
	64 + 32 + 16 + 0 + 0 + 2 + 1 = 115						
	64	32	16	8	4	2	1
	1	1	0	1	1	1	0
	64 + 32 + 0 + 8 + 4 + 2 + 0 = 110						
115 – 110 = 5 And 5 in base 2 is 101₂							
b)				125	25	5	1
				4	0	3	1
				500 + 0 + 15 + 1 = 516			
				125	25	5	1
				4	1	1	4
				500 + 25 + 5 + 4 = 534			
516 + 534 = 1050 And 1050 in base 5 is 13200₅							

Prime Numbers and Number bases:

A **prime number** is a number that is divisible by only two numbers, one and itself. Prime numbers are also an important concept and when well founded and understood learners will not experience problems with factorising. Upon conducting an investigation, Ferguson found an interesting pattern with prime numbers. The table below shows 36 counting numbers arranged in rows of 6. The first column and fifth have most of the numbers as prime numbers except 2 and 3 which are not in those column.

Base Ten					
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60

Table 4: Counting Numbers arranged in 6 rows

When the same numbers are written in base 6, the pattern is more distinct where the first and fifth column have a general form $6n \pm 1$.

$$7, 13, 19, \dots 6n + 1$$

$$5, 11, 17, \dots 6n - 1$$

Base Six					
1	2	3	4	5	10
11	12	13	14	15	20
21	22	23	24	25	30
31	32	33	34	35	40
41	42	43	44	45	50
51	52	53	54	55	100
101	102	103	104	105	110
111	112	113	114	115	120
121	122	123	124	125	130
131	132	133	134	135	136

Table 5: Numbers in base 6 arranged in 6 rows

Learners can work on patterns in a similar way.

Activity 5:

- a) Arrange counting numbers in rows of 4 up to 40. Mark all the prime numbers. Which rows have the most prime numbers?
- b) Change the numbers above to base four.
- c) Find the general form of the sequences with most prime numbers.

[5 min]

Further:

Activity 6:

- a) Arrange counting numbers in rows of 12 up to 60. Mark all the prime numbers. Which rows have the most prime numbers?
- b) Change the numbers above to base twelve.
- c) Find the general form of the sequences with most prime numbers.

[5 min]

In conclusion, learners who have missed the grasp of key basic concepts need to catch-up in order to progress faster with higher concepts. Activities given above can be added as intervention for the catch-up.

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